Variation of Relative Temperature at the Surface and at the Center of a Plate
($Sk = 0.5$; $Bi = 1.0$; $\Theta_0 = 0.2$)

Fo	Bi_1^*	Θ_1 (1, Fo)	Bi_2^* (Fo)	Θ_2 (1, Fo)	Θ(1, Fo) from [1]	Θ_2 (0, Fo)	Θ(0, Fo) from [1]
0	1.624	0.2	1.624	0.2	0.2	0.2	0.2
0.1	1.624	0.512	1.954	0.552	0.553	0.201	0.209
0.2	1.624	0.586	2.065	0.636	0.644	0.255	0.262
0.3	1.624	0.633	2.144	0.687	0.699	0.328	0.336
0.4	1.624	0.671	2.212	0.728	0.742	0.401	0.409
0.5	1.624	0.704	2.274	0.763	0.778	0.467	0.478
0.6	1.624	0.733	2.331	0.792	0.808	0.528	0.540
0.8	1.624	0.782	2.436	0.842	0.856	0.632	0.646
1.0	1.624	0.822	2.527	0.880	0.893	0.714	0.729
1.2	1.624	0.855	2.606	0.909	0.919	0.779	0.793
1.6	1.624	0.904	2.729	0.948	0.955	0.870	0.881
2.0	1.624	0.936	2.816	0.971	0.975	0.924	0.932

it is more acceptable for the case of relatively large Sk and Bi that the inverse phenomenon will occur. This is due to the fact that then

$$Bi^*(Fo) \leqslant Bi_2^*(Fo) \leqslant Bi_1^*.$$

The method described may be used for bodies of different geometrical configuration (cylinders, spheres, prisms, etc.), and also for other nonlinear boundary conditions.

NOTATION

$\Theta(X, Fo) = T(X, Fo)/T_m$ is the relative temperature; T_m is the temperature of the medium; T_0 is the initial temperature; δ is the plate half width; α is the heat transfer coefficient; σ_v is the view factor for radiative heat transfer; a is diffusivity; τ is time; $\Theta_0 = T_0/T_m$; $Fo = a\tau/\delta^2$; $Bi = \alpha\delta/\lambda$; $Sk = \sigma_v T_m^3 \delta/\lambda$.

REFERENCES

1. Yu. V. Vidin and V. V. Ivanov, Calculation of Temperature Fields in Solid Bodies Heated by Simultaneous Convection and Radiation [in Russian], Krasnoyarsk Polytechnic Institute, 1965.

2. A. V. Luikov, Theory of Heat Conduction [in Russian], Gostekhizdat, 1952.

19 July 1966

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HEAT TRANSFER IN LAMINAR FLOW OF AN INCOMPRESSIBLE FLUID IN A ROUND TUBE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 672-674, 1967

UDC 536.242

In the solution of this problem it is usually assumed that heat conduction in the direction of flow is negligibly small in comparison with convective heat transfer. When this assumption is made and when thermophysical characteristics are assumed to be constant and the velocity profile across the tube to be parabolic (which corresponds to steady rectilinear symmetric isothermic laminar flow), the first boundary-value problem can be formulated as follows:

$$\frac{\partial^2 t}{\partial R^2} + \frac{1}{R}\frac{\partial t}{\partial R} = (1 - R^2)\frac{\partial t}{\partial Z},$$

$$0 \leqslant R \leqslant 1, \quad 0 \leqslant Z < +\infty, \tag{1}$$

$$t(0, Z) < +\infty, \tag{2}$$

$$t(1, Z) = f(Z), \tag{3}$$

$$t(R, 0) = \varphi(R). \tag{4}$$

To solve this problem we first solve the corresponding homogeneous problem, viz., Eq. (1) on condition that on the surface of the tube

$$t(1, Z) = 0. \tag{5}$$

We will seek special solutions of this auxiliary problem in the form of products $M(R)\exp(-\mu^2 Z)$ on condition that $M(R)$ is the solution of the following Sturm-Liouville problem [1]:

$$\frac{d^2M}{dR^2} + \frac{1}{R}\frac{dM}{dR} + \mu^2(1 - R^2)M = 0,$$

$$M(0) < +\infty, \quad M(1) = 0. \tag{6}$$

Direct substitution shows that the solution of problem (6) will be a function

$$T(\mu R^2) = F(a, 1, \mu R^2)\exp\left(-\frac{\mu}{2}R^2\right), \tag{7}$$

where $F(a, 1, \mu R^2)$ is a degenerate hypergeometric function, and $a = (2 - \mu)/4$.

Expressing the exponential and hypergeometric function in the form of power series in R^2 [2-4] and multiplying these series, which is possible in view of their absolute convergence, we obtain

$$T(\mu R^2) = 1 + \sum_{k=1}^{\infty}\left(\frac{\mu}{2}\right)^k \times$$

$$\times R^{2k}\sum_{s=0}^{k}(-1)^{s+1}\frac{2^s\,\Gamma(a+s)}{\Gamma(a)(s!)^2(k-s)!}. \tag{8}$$

To obtain nontrivial solutions we must find the eigenvalues of μ from the equation

$$F\left(\frac{2-\mu}{4},\ 1,\ \mu\right)=0, \tag{9}$$

which can be solved graphically.

The corresponding eigenfunctions in the interval [0, 1] $T(\mu R^2)$ will be orthogonal with a weight $R(1 - R^2)$ [5], i.e., the following equality will be valid:

$$\int_0^1 R\,(1-R^2)\,T\,(\mu_n R^2)\,T\,(\mu_m R^2)\,dR = \begin{cases} 0, & n \neq m \\ \left[\dfrac{dF\,(a_n,\ 1,\ \mu_n)}{dR}\exp\left(-\dfrac{\mu_n}{2}\right)\right]^2, & n = m. \end{cases} \tag{10}$$

Following Grinberg's method [6], we will seek the solution of problem (1) with boundary conditions (2), (3), (4) in the form

$$t=\sum_{i=0}^{\infty}\frac{t_i\,(Z)}{\left[\dfrac{dF\,(a_i,\ 1,\ \mu_i)}{dR}\exp\left(-\dfrac{\mu_i}{2}\right)\right]^2}\,T\,(\mu_i R^2), \tag{11}$$

where

$$t_i\,(Z)=\int_0^1 t\,(R,\ Z)\,R\,(1-R^2)\,T\,(\mu_i R^2)\,dR.$$

Substituting (11) in (1) and using the boundary conditions we obtain the following ordinary linear differential equation of the first order to determine $t_i(Z)$:

$$\frac{dt_i}{dZ}+\mu_i^2\,t_i\,(Z)+k_i f\,(Z)=0, \tag{12}$$

where

$$k_i=2\mu_i\,\frac{dF\,(a_i,\ 1,\ \mu_i)}{dR}\exp\left(-\frac{\mu_i}{2}\right),$$

the solution of which will be

$$t_i=\exp\,(-\mu_i^2 Z)\,[C_i-k_i\int f\,(Z)\exp\,(\mu_i^2 Z)\,dZ]. \tag{13}$$

The constant C_i is determined from the boundary condition (4) by multiplying this condition by the eigenfunction and integrating over the interval [0, 1]. For this we need to know the forms of functions $f(Z)$ and $\varphi(R)$ and for the latter, as a rule, we need to assume that it can be expanded as a power series. In particular, when $\varphi(R) \equiv 0$, $f(Z) \equiv f_0 = \text{const}$

$$C_i=k_i\,\frac{f_0}{\mu_i^2}\,.$$

We denote the found values of C_i by C_i^0, and then the solution of our problem will take the form (11), where t_i is given by formula (13) with $C_i = C_i^0$.

The problem is solved in exactly the same way with a boundary condition of the second kind, i.e., if condition (3) of our problem is replaced by the condition

$$\frac{\partial t\,(1,\ Z)}{\partial R}=f\,(Z). \tag{14}$$

In this case the eigenvalues will be given by the following system:

$$2aF\,(a+1,\ 2,\ \mu)=F\,(a,\ 1,\ \mu),$$
$$a=(2-\mu)/4. \tag{15}$$

NOTATION

$R \equiv r/r_0$ is the dimensionless variable radius; $Z \equiv z/r_0\mathrm{Pe}$ is the reduced tube length; $\mathrm{Pe} = 2w_{av}r_0/a$ is the Peclet number; r_0 is the tube radius; r and z are cylindrical coordinates; w_{av} is the average flow velocity; $\Gamma(a)$ is the gamma function; t is the temperature of the liquid.

REFERENCES

1. E. L. Ince, Ordinary Differential Equations [in Russian], GONTI, 1938.
2. N. N. Lebedev, Special Functions and Their Applications [in Russian], Gostekhizdat, 1961.
3. D. S. Kuznetsov, Special Functions [in Russian], Izd. Vysshaya shkola, Moscow, 1965.
4. V. I. Smirnov, Course of Higher Mathematics, Vols. 2 and 3, Gostekhizdat, 1964.
5. N. S. Koshlyakov, E. B. Gliner, and M. M. Smirnov, Fundamental Differential Equations of Mathematical Physics [in Russian], Fizmatgiz, 1962.
6. G. A. Grinberg, Selected Questions of Mathematical Theory of Electric and Magnetic Phenomena [in Russian], Izd. AN SSSR, 1948.

2 July 1966
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THERMAL CONDUCTIVITY OF SOME METALS AND ALLOYS IN THE TEMPERATURE RANGE 4.2–273° K

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Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 5, pp. 675–677, 1967
UDC 536.512

In the thermal potentiometer method of measurement of thermal conductivity of metals (Fig. 1) [1], the total heater power may be represented in the form

$$W=W_1+W_2,$$

where W_1 is the power passing through the cross section of the specimen T_1; and W_2 is the power scattered by radiation from the specimen surface S_1, located below the section T_1.

Similarly

$$W_1=W_3+W_4,$$

where W_4 is the power passing through the specimen cross section T_2; and W_3 is the power scattered by radiation from the specimen surface located between sections T_1 and T_2.

For $T_1 - T_0 \approx 1°$, W_3 is not less than 1% of W, right up to 100-150° K. W_4 is a quantity of the second order of smallness in compari-